Instructors: **1.** Dr. Rola Alseidi	**Philadelphia University** **Faculty of Science** **Department of Mathematics** **Midterm Exam**	**Academic Year:** 2022-2023 **Semester:** Fall **Date:** 7/12/2022 **Course:** Matrix Analysis **Duration:** 75 Min

Name: **I.D. Number:**

Question One: [3 points]

If $A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$ and $\det(A) = 4$. Find Let V be the set of positive real numbers and define the operations on V to be

$$u + v = uv, ku = u^k, k \in \mathbb{R}$$

(a) (1 point) Find $3 + 2$.

(b) (1 point) What is the zero vector in this space.

1. $\det(-2A)^{-1}$

2. $\det\left(\begin{bmatrix} g & h & i \\ a & b & c \\ -d & -e & -f \end{bmatrix}\right)$

Question Two: [8 points]

For the matrix $A = \begin{bmatrix} 0 & 3 \\ 2 & -1 \end{bmatrix}$. Find

1. The eigenvalues of the matrix A.

2. The eigenvectors of the matrix A.

3. The matrix P that diagonalizes A.

4. A^{10}.

5. What is the dominant eigenvalue of A , explain your answer.

Question Three: [4 points]

Use the power method to approximate the dominant eigenvalue of A, use the given initial vector x_0, the specified number of iterations is 3 and three decimal accuracy.

$$A = \begin{bmatrix} -6 & 4 \\ 8 & -2 \end{bmatrix}, x_0 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, k = 4$$

Question Four: [4 points]

Given the following vectors form a basis for subspace W of $\mathbb{R}^4$. Apply Gram-Schmidt process to obtain an orthonormal basis for W.

$$, x_1 = \begin{bmatrix} 1 \\ 2 \\ -2 \\ 1 \end{bmatrix} \; x_2 = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 2 \end{bmatrix}, \; x_3 = \begin{bmatrix} 1 \\ 8 \\ 1 \\ 0 \end{bmatrix}$$

Question Five: [4 points]

If $A \in M_n(\mathbb{R})$ with eigenvalues $\lambda_1, \lambda_2, \ldots, \lambda_n$, prove that $\det(A) = \prod_{i=1}^{2} \lambda_i$

Question Six: [4 points] Circle True or False. Read each statement carefully before answering.

(a) True False If A is diagonalizable, then there is a unique invertible matrix P such that $P^{-1}AP$ is diagonal.

(b) True False The null space of 3×4 matrix always has dimension at least one.

(c) True False If V is a 5-dimensional vector space, then every collection of five vectors in V is linearly independent.

(d) True False Let A be 3×3 matrix with rank 2, then the homogeneous system $Ax = 0$ has infinity many solutions.

Question Seven: [3 points] Circle the correct answer

(a) Which of the following sets of vectors in $\mathbb{R}^2$ is not a base

A. $S = \{(2,7),(4,5)\}$ B. $S = \{(2,5),(1,5)\}$ C. $S = \{(0,0),(2,8)\}$
D. $S = \{(1,2),(1,4)\}$ E. $S = \{(2,0),(0,2)\}$ F. None

(b) Let A be a 6×6 matrix with the characteristic equation

$$\lambda^2(\lambda - 1)(\lambda - 2)^3 = 0.$$

The algebraic multiplicity of $\lambda = 2$ is

A. 2 B. ≤ 2 C. 1 D. ≤ 1 E. 3 F. ≤ 3

(c) If the characteristic polynomial for a matrix A is

$$p(\lambda) = (1 - 2\lambda)(\lambda - 1)^2.$$

Then $tr(2A) - \det(A^3) =$

A. $\frac{25}{8}$ B. $\frac{11}{2}$ C. $\frac{9}{2}$ D. $\frac{39}{8}$ E. $\frac{41}{8}$
F. None

Good Luck